**Theories**

* **Hidden premises**
* E.g. Ottawa is north of Toronto ∧ Toronto is north of Waterloo ⇒ Ottawa is north of Waterloo
  + Hidden premise – “north of” is transitive
* **Enthymeme** – an argument that contains hidden premises – contains unstated premises that are obviously true
* Ex:
  + “The crime was committed by someone at the general store at 6pm”
  + “Billy the Kid was in the jail at 6pm”
  + “Therefore, Billy the Kid did not commit the crime at the general store at 6pm”
  + cc(x, y, z) means x committed a crime at location y at time z
  + loc(x, y, z) means x was at location y at time z
  + GS = general store
  + B = Billy
  + J = jail
  + 1) ∃x . cc(x, GS, 6)
  + 2) location(B, J, 6)
  + |= ¬cc(B, GS, 6)
  + hp1) ∀x, y, z, w . loc(x, y, w) ∧ loc(x, z, w) ⇒ y = z
  + hp2) ∀x, y, z . cc(x, y, z) ⇒ loc(x, y, z)
  + hp3) ¬(J = GS)
  + 3) disprove cc(B, GS, 6)
    - 4) cc(B, GS, 6) ⇒ loc(B, GS, 6) by forall\_e on hp2
    - 5) loc(B, GS, 6) by imp\_e on 3, 4
    - 6) loc(B, J, 6) ∧ loc(B, GS, 6) ⇒ J = GS by forall\_e on hp1
    - 7) loc(B, J, 6) ∧ loc(B, GS, 6) by and\_i on 2, 5
    - 8) J = GS by imp\_e on 6, 7
    - 9) false by not\_e on 3, 8
  + 10) ¬cc(B, GS, 6) by raa on 3-9
* **Theory** – a set of axioms (facts) about specific constants, functions, and predicate symbols
  + Also the set of all the theorems provable from those axioms
  + An axiom is an inference rule that has no premises
  + Ex: Harry is John’s brother; John is Bill’s brother
    - br(x, y) means x and y are brothers
    - 1) br(H, J) premise
    - 2) br(J, B) premise
    - t1) ∀x, y, z . br(x, y) ∧ br(y, z) ⇒ br(x, z)
    - 3) br(H, J) ∧ br(J, B) ⇒ br(H, B) by forall\_e on t1
    - …
    - Interpretation:
      * D = {Bill, John, Harry}
      * B → Bill, J → John, H → Harry
      * br(., .) → brother(Harry, John) := T etc. …
    - But there can be other interpretations where the axiom holds true (when it might not be intended to be)
  + Model of the theory – an interpretation in which all the axioms of the theory are true
    - One such model is seen as the standard/normal interpretation of the theory
    - Set of axioms must be consistent for the proof to be valid
* **Predicate logic with equality**
  + Equality – predicate symbol “=”
    - A binary infix predicate – arguments must be terms
    - a = b is an atomic well-formed formula
    - Note associativity does not apply to equality
      * a = b = c → a = (b = c) → b = c returns a truth value; doesn’t make sense
  + Ex: “Rima’s age is 5”
    - Age(Rima) = 5
  + “Five is prime”
    - Prime(5) – not an equality
  + “There exists at least one solution”
    - ∃x . sol(x)
  + “There is exactly one solution”
    - ∃x . sol(x) ∧ ∀y . sol(y) ⇒ x = y
  + “There is at most one solution”
    - ∀x, y . sol(x) ∧ sol(y) ⇒ x = y
  + “Alice likes only bubble gum ice cream”
    - likes(Alice, BGIC) ∧ (∀x . likes(Alice, x) ⇒ x = BGIC)
  + “Only Alice likes bubble gum ice cream”
    - ∀x . likes(x, BGIC) ⇔ x = Alice
  + “The only kind of ice cream Alice likes is bubble gum”
    - ∀x . icecream(x) ⇒ (likes(Alice, x) ⇔ x = BGIC)
  + Natural deduction rules:
    - Reflexivity: t = t
    - Substitution:
      * t1 = t2, P[t2/x] → P[t1/x]
      * t1 = t2, P[t1/x] → P[t2/x] where t1, t2 are free for x in P
  + Derived properties from these 2 rules:
    - Symmetry:
      * ∀x, y . (x = y) |− y = x
    - Transitivity
      * ∀x, y, z . (x = y) ∧ (y = z) |− (x = z)
  + Leibniz’s Law (equals for equals):
    - t1 = t2 |− P(t1) ⇔ P(t2)
  + Normal interpretations – interpretations in which ‘=’ is interpreted as equality on the objects of the domain